Realistic Symmetries: How They Determine the Accuracy of Interferometry + Polarimetry for Characterizing Vegetated Land Surfaces

Robert N. Treuhaft

Jet Propulsion Laboratory, 300-227
California Institute of Technology
4800 Oak Grove Drive
Pasadena, CA 91109 USA
Phone/Fax: 818-354-6216/818-393-5285; email: rnt@radar-sci.jpl.nasa.gov

Abstract

This paper suggests an approach to combining radar interferometry and polarimetry for vegetated land surfaces. The approach is based on quantitative parameter estimation, starting with simple, very symmetric physical models relating the radar observations to vegetation parameters. Model asymmetry is increased to decrease systematic model error, with a corresponding increase in the number of model parameters, and consequent increase in statistical parameter error. Models with realistic symmetries optimize the balance between statistical and systematic error.

1 Introduction

Interferometry is mainly sensitive to the spatial distribution of vegetation scatterers [1], while polarimetry is primarily sensitive to scatterer shape and orientation [2, 3]. Combining radar interferometry and polarimetry, including polarimetric interferometry, can improve the estimation of parameters describing vegetated land surfaces, in which spatial and shape-orientation characteristics are correlated. Because interferometric and polarimetric radar usually constitute a small (≈ 10) number of observations per resolution cell, the number of parameters which can be estimated from radar measurements is correspondingly small. Simple physical models are required for quantitative estimation in order to keep the parameter set small. Schematically, a physical model $\mathbf{M}$ follows from basic electromagnetic scattering theory and statistics, and is used to estimate a parameter vector $\underline{P}$, from a radar observation vector $\underline{O}$, containing both interferometric and polarimetric data:

$$\underline{P} = \mathbf{M}^{-1}\underline{O} \tag{1}$$

Symmetries must be assumed in the physical model $\mathbf{M}$ in order to keep the number of parameters less than or equal to the number of observations. A symmetry is a statement of the invariance of the target under some operation, usually translation or rotation. The following sections present an approach to combined interferometric and polarimetric analysis, giving two physical models as examples of relating observations $\underline{O}$ to parameters $\underline{P}$. A possible path for research into interferometric polarimetric analysis begins with a very simple physical model. Comparing estimated parameters to ground truth then guides increases in model complexity until the systematic errors are below the statistical errors, achieving "optimal parameter estimate accuracy."

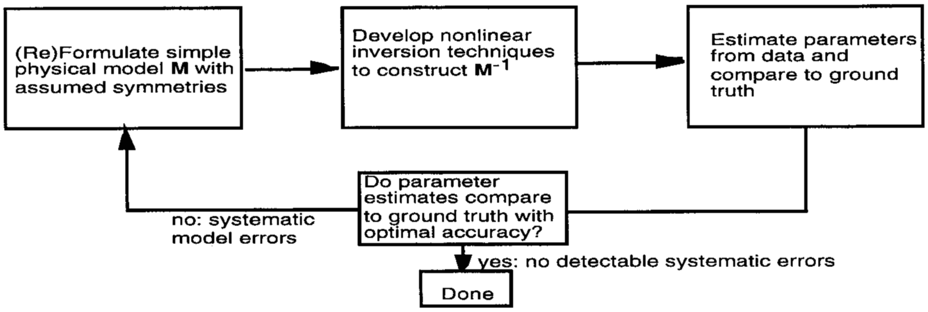

Figure 1: A path for research into interferometric, polarimetric analysis.

The increasingly more complicated reformulation of the model in Figure 1 is generally achieved by introducing asymmetries, with the usual consequence of increasing the parameter set, and the byproduct of increasing statistical estimation errors. In order to illustrate the approach in Figure 1, the next two sections describe two candidate physical models (two loops through Figure 1), their parameters, and the role of interferometry and polarimetry.

2 A Homogeneous Randomly Oriented Volume

A simple model based on randomly oriented volumes is a legitimate starting point for many vegetation types, for example, forests. For a randomly oriented volume, the invariance of the average target scatterer orientation is assumed under rotation. Homogeneity implies that vegetation statistical characteristics are invariant under translation within the vegetation. Because of the rotational symmetry, polarimetry does not help and the interferometric cross correlation amplitude and phase can be expressed in terms of three parameters [1]:

$$\begin{pmatrix} \text{Vegetation Height } (h_v) \\ \text{Surface Height } (z_0) \\ \text{Extinction Coefficient } (\sigma_x) \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} \text{Interferometric Amplitude } (A_{1V}) \\ \text{Interferometric Phase } (\phi_{1V}) \\ \text{Interferometric Amplitude } (A_{2V}) \\ \text{Interferometric Phase } (\phi_{2V}) \end{pmatrix} \tag{2}$$

where two baselines (4 observations) on the right are needed to estimate the 3 parameters on the left. In Eq. (2), A_{1V} and ϕ_{1V} are the interferometric amplitude and phase from the first baseline, which transmits and receives at V-polarization at both ends. If the symmetries assumed in the homogeneous, randomly-oriented-volume model are realistic, then with two baselines, one loop through Figure 1 will be sufficient.

3 Randomly Oriented Volume Plus a Horizontal Ground Surface

It may well be (e.g. [4]) that the vegetation height parameters, for example, estimated from a given area, when compared to ground truth, are found to exhibit systematic error due to the oversimplification of the assumed randomly-oriented-volume symmetries. Adding a horizontal ground-volume bounce contribution (which includes ground-trunk) in the physical model adds a ground scattering component with a preferred orientation (horizontal), at a preferred location (at the bottom of the vegetation layer). Adding the ground asymmetry requires adding an additional parameter, Δ_V. This parameter contains the product of the squared ground specular reflection coefficient at the incident-receive polarization (assumed to be V) and the ratio of the squared specular- to back-scattering amplitudes of the volume [5]. The 2-baseline interferometric parameter estimation can now be written as

$$\begin{pmatrix} h_v \\ z_0 \\ \sigma_x \\ \Delta_V \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} A_{1V} \\ \phi_{1V} \\ A_{2V} \\ \phi_{2V} \end{pmatrix} \tag{3}$$

If the polarimetric $\{HHHH/VVVV\}$ ratio were also measured, another parameter Δ_H including the H-polarization ground reflection coefficient must be added to describe the ground asymmetry, yielding:

$$\begin{pmatrix} h_v \\ z_0 \\ \sigma_x \\ \Delta_V \\ \Delta_H \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} A_{1V} \\ \phi_{1V} \\ A_{2V} \\ \phi_{2V} \\ \{HHHH/VVVV\} \end{pmatrix} \tag{4}$$

Fully polarimetric interferometry supplies more observations, and adds no new parameters in the following possible estimation scenario:

$$\begin{pmatrix} h_v \\ z_0 \\ \sigma_x \\ \Delta_V \\ \Delta_H \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} A_{1V} \\ \phi_{1V} \\ A_{2V} \\ \phi_{2V} \\ A_{1H} \\ \phi_{1H} \\ A_{2H} \\ \phi_{2H} \\ \{HHHH/VVVV\} \end{pmatrix} \tag{5}$$

Eq. (5) suggests that interferometric amplitudes and phases from one fully polarimetric baseline (e.g. A_{1V}, ϕ_{1V}, A_{1H}, and ϕ_{1H}) plus the $\{HHHH/VVVV\}$ ratio might be sufficient to determine the parameters required to describe a random volume over a horizontal surface. This is an example of the general point that fully polarimetric interferometry strengthens parameter estimation when the vegetated land surface contains oriented components. In fact, single-baseline solutions for vegetation height, topography, and extinction coefficient may be enabled [6].

If the parameter estimates resulting from Eq. (5) exhibit intolerable systematic error, it will be necessary to investigate orientation and distribution effects in both the volume and the surface (another loop through Figure 1), again reducing the level of assumed symmetry, with a possible increase in parameters. Polarimetric interferometric amplitude optimization techniques [7] may help to reduce the number of parameters that must be determined, and thereby play an important role in enabling nonlinear parameter estimation.

4 Conclusion

An approach to the combination of interferometry and polarimetry involves assuming very symmetric models and increasing the level of asymmetry until parameter estimates describing vegetated land surfaces exhibit optimal accuracy. The application of interferometry, polarimetry, and polarimetric interferometry to random volumes and horizontal ground surfaces demonstrates the increased parameter set needed when model asymmetry increases. Research in this field must determine levels of realistic symmetry that can be used in models for data analysis, in order to balance systematic and statistical error and obtain optimally accurate parameter estimates.

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References

[1] R. N. Treuhaft, S. N. Madsen, M. Moghaddam, and J. J. van Zyl, "Vegetation Characteristics and Surface Topography from Interferometric Radar," *Radio Science*, vol. 31, pp. 1449-1485, December 1996.

[2] J. J. van Zyl, H. A¿ Zebker, and C. Elachi, Imaging radar polarization signatures: Theory and observations, *Radio Science*, vol. 22, pp. 529-543, 1987.

[3] W. M. Boerner, W. M., El-Arini, M. B., Chan, C. Y., Mastoris, P. M., "Polarization Dependence in Electromagnetic Inverse Problems," *IEEE Trans. Ant. and Prop.*, **AP-29**, 262-271, 1981.

[4] R. N. Treuhaft and M. Moghaddam, "A Unified Analysis of Radar Interferometry and Polarimetry for the Estimation of Forest Parameters," *IGARS98*, Seattle, 1998.

[5] R. N. Treuhaft, "The Vertical Structure of Vegetated Land Surfaces from Interferometric and Polarimetric Radar," in preparation, 1998.

[6] R. N. Treuhaft and S. R. Cloude, "Vegetation Structure from Polarimetric Interferometry," in preparation, 1998.

[7] S.R. Cloude and K. P. Papathanassiou, "Polarimetric SAR Interferometry", *IEEE Trans. Geosci. Remote Sensing*, in press, 1998.